String Theory in a Nutshell, 2^{nd} edition.

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Abstract

These are partial solutions for the textbook, String Theory in a Nutshell by Elias Kiritsis. Due to the number of questions that are at the end of each chapter, only the problems that have been deemed *important*. Before using these solutions, or referencing to them, please double-check them, and if an error or misunderstanding of the problem is found, please email me. Since this textbook, and subject in general is quite **dense**, the solutions for this textbook may be slow. To ensure that the solutions are correct (or at least *quite* close), they will be cross-checked with Polchinski's text on string theory. Also, I may not go through each chapter, since they may not pertain to what I want to conduct research in. These solutions are uploaded on my website: alexcassem.net on the blog tab.

Contents

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1 Chapter 1: Introduction

There are no exercises in chapter 1. This chapter is an historical introduction into the field, and the motivation that eventually lead to formulating string theory.

2 Chapter 2: Classical String Theory

Exercise 2.2: Use ζ-function regularization of the determinant in equaiton (2.1.26 on page 15) to produce (2.1.27)

Solution: The determinant in question is,

$$det\left(-\frac{1}{L^2}\partial_\tau^2\right) = \prod_{n=1}^{\infty}\frac{n^2}{L^2}. \tag{1}$$

So, the question is asking use to regularize the denominator as $L^{-2} \rightarrow L$ and the numerator to become, $n^a \rightarrow (2\pi)^{a/2}$. This is done by using the ζ-function which is,

$$\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{\Gamma(s)}\int_0^\infty \frac{x^{s-1}}{e^x - 1}dx, \tag{2}$$

where $\Gamma(s) = \int_0^\infty x^{s-1}e^{-x}dx$. From this, the L^{-2} term becomes $\prod_{n=1}^{\infty} L^{-2} = L^{-2\zeta(0)}$, thus in setting $s = 0$ we must evaluate,

$$\begin{aligned}\zeta(0) &= \sum_{n=1} \rightarrow \\ &= \frac{1}{\Gamma(0)}\int_0^\infty \frac{x^{-1}}{e^x - 1}, \\ &= -\frac{1}{2\Gamma(1)}\int_0^\infty sech^2(t)dt = -\frac{1}{2} + 2sin(0)\int_0^\infty \frac{dt}{e^{2\pi t}-1} = -\frac{1}{2}\end{aligned} \tag{3}$$

So, this gives the correct value when regularizing L (as seen in equation 2.1.27) as, $L^{-2\zeta(0)} = L^{-2(-1/2)} = L$. We can do the same thing for when $a = 2$ being,

$$\begin{aligned}\prod_{n=1}^{\infty} n^a &= e^{-a\zeta'(0)}, \\ \zeta'(s) &= -\sum_{k=1}\frac{ln(k)}{k^s} = -\sum ln(k) = -\frac{1}{2}ln(2\pi), \\ \prod_{n=1}^{\infty} n^a &= e^{-a(-1/2ln(2\pi))} = e^{a/2ln(2\pi)} = (2\pi^{a/2}),\end{aligned} \tag{4}$$

which is exactly what is shown in equation 2.1.27. A lot of help in solving this problem came from *Wolfram's MathWorld* via their online database on the ζ-function which can be found here.

Exercise 2.3: Consider the worldline action of a point particle in an arbitrary spacetime metric $G_{\mu\nu}$. Derive the equations of motion for the path, and show that these are equivalent to the geodesic equations.

Solution: The worldine for such a point particle is just,

$$S = -\int ds = -m\int\sqrt{-G_{\mu\nu}dx^\mu dx^\nu} = -m\int\sqrt{-G_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}}ds. \tag{5}$$

It is apparent that our Lagrangian is, $\mathcal{L} = \sqrt{G_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}$, with $\dot{x}^\mu = dx^\mu/ds$. We can use the Euler-Lagrange equations,

$$\frac{\partial}{\partial s}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}^\mu}\right) - \frac{\partial\mathcal{L}}{\partial x^\mu} = 0, \tag{6}$$

in order to find the equations of motion, which *should* yield the geodesic equations. As a note, the our general metric of course has dependence upon the spacetime coordinates, $G_{\mu\nu}(x^\lambda(s))$. The first part we will evaluate is the "velocity" derivative,

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\dot{x}^\alpha} &= \frac{\partial}{\partial\dot{x}^\alpha}\left(\sqrt{G_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}\right), \\
&= \frac{1}{2\sqrt{G_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}}\left(-G_{\mu\nu}\delta^\mu_\alpha\dot{x}^\nu - G_{\mu\nu}\dot{x}^\mu\delta^\nu_\alpha\right) \\
&= \frac{1}{2\mathcal{L}}\left(-G_{\mu\alpha}\dot{x}^\mu - G_{\mu\alpha}\dot{x}^\mu\right) \\
&= -\frac{G_{\mu\alpha}}{\mathcal{L}}\frac{dx^\mu}{ds},
\end{aligned} \tag{7}$$

however, we can get rid of $\mathcal{L}$ by recalling the definition of the action, $s = \int\mathcal{L}d\tau \to \mathcal{L} = ds/d\tau$, which when plugged into the above gives,

$$= -G_{\mu\alpha}\frac{dx^\mu}{ds}\frac{ds}{d\tau} = -G_{\mu\alpha}\frac{dx^\mu}{d\tau}. \tag{8}$$

The second part will be the derivative with respect to "position" giving,

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial x^\alpha} &= \frac{1}{2\mathcal{L}}\frac{\partial}{\partial x^\alpha}\left(G_{\mu\nu}\right)\dot{x}^\mu\dot{x}^\nu \\
&= -\frac{1}{2\mathcal{L}}\frac{\partial G_{\mu\nu}}{\partial x^\alpha}\frac{dx^\mu}{ds}\frac{ds}{d\tau}\frac{dx^\nu}{ds} \\
&= -\frac{1}{2}\frac{\partial G_{\mu\nu}}{\partial x^\alpha}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}.
\end{aligned} \tag{9}$$

Now, when we plug everything into the Euler-Lagrange equations will yield us,

$$\begin{aligned}
&\frac{\partial}{\partial s}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}^\mu}\right) - \frac{\partial\mathcal{L}}{\partial x^\mu} = 0 \to \\
&\frac{d}{d\tau}\left(G_{\mu\alpha}\frac{dx^\mu}{d\tau}\right) + \frac{1}{2}\frac{\partial G_{\mu\alpha}}{\partial x^\alpha}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0.
\end{aligned} \tag{10}$$

The proper time derivative will give,

$$\begin{aligned}
\frac{d}{d\tau}\left(G_{\mu\alpha}\frac{dx^\mu}{d\tau}\right) &= G_{\beta\alpha}\frac{d^2x^\mu}{d\tau^2} + \frac{dx^\beta}{d\tau}\frac{\partial}{\partial\tau}\left(G_{\beta\alpha}(x^\gamma)\right) \\
&= G_{\beta\alpha}\frac{d^2x^\beta}{d\tau^2} + \frac{\partial G_{\alpha\beta}}{\partial x^\gamma}\frac{dx^\gamma}{d\tau}\frac{dx^\beta}{d\tau},
\end{aligned} \tag{11}$$

where we renamed the index $\mu \to \beta$ to not confuse the dummy-summation indexes. After we put this term together with that in equation 10 and simplify gets us,

$$\frac{d^2 x^\gamma}{d\tau^2} + G^{\gamma\alpha}\left[\partial_\mu G_{\alpha\nu} - \frac{1}{2}\partial_\alpha G_{\mu\nu}\right]\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0. \tag{12}$$

This *is* the geodesic equation, but it may appear unrecognizable since we don't find the typical Christoffel connection on the second term. But, if you wanted to recover the explicit form, split the second term in the square brackets with the 1/2 term into two halves. Then, change the sumation signs from $\gamma \to \sigma$ and $\beta \to \mu$ for the new summations, then multiply by $G^{\sigma\mu}$ to recover the explicit Christoffel connection, $\Gamma^\gamma_{\mu\nu}$.

Exercise 2.6: Consider a point particle of charge e moving in a nontrivial metric and an electromagnetic potential A_μ. Show that the electromagnetic coupling is described by the addition of,

$$\Delta S = e\int d\tau A_\mu \dot{x}^\mu, \tag{13}$$

to the action. Derive the equations of motion. If one coordinate is cyclic, derive the associated conserved momentum.

Solution: There are a few equations we need. First, the equation of motion for a point particle (last question), the conserved momentum, $p_\mu = \partial\mathcal{L}/\partial\dot{x}\mu$, and the constraint equation, $p^\mu p_\mu + m^2 = 0$.

The first part of the question is asking that the electromagnetic coupling is described by the addition of equation 13, but this is slightly obvious once we find the equations of motion for a charged particle, and then find the momentum and show what the mass term is. But, to show this explicitly, you can check the Lorentz invariance. So, skipping to finding the equations of motion, the entire action is,

$$S = -m\int\sqrt{-G_{\mu\nu}dx^\mu dx^\nu} + e\int A_\mu \dot{x}^\mu, \tag{14}$$

where A_μ is the electromagnetic potential that is approximately, $A\mu \sim (E, B^i)$. Now, we find the equations of motion. The first part is just the geodesic equations, which we just found. So, using the Euler-Lagrange equations will only change on the "velocity" term giving,

$$\frac{\partial\mathcal{L}}{\partial\dot{x}^\alpha} = -G_{\mu\alpha}\frac{dx^\mu}{d\tau} + eA_\mu\delta^\mu_\alpha. \tag{15}$$

Then with equation 8 from the previous question, then entire equation of motion will be,

$$\frac{d}{d\tau}\left[\frac{1}{2}\partial_\alpha G_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\right] + G_{\mu\alpha}\dot{x}^\mu - eA_\alpha = 0. \tag{16}$$

From this, we can directly compute the momentum with respects to $\dot{x}^\alpha$ giving,

$$p_\alpha = \frac{\partial\mathcal{L}}{\partial\dot{x}^\alpha} = -G_{\mu\alpha}\frac{dx^\mu}{d\tau} + eA_\alpha. \tag{17}$$

This momentum should obey our constraint equation, so lets check,

$$\begin{aligned} &p^\mu p_\mu + m^2 = 0 \\ &(G_{\mu\alpha}\dot{x}^\mu + eA_\alpha)(-G^{\mu\alpha}\dot{x}_\mu + eA^\alpha) + m^2 = 0 \\ &|G| - e^2|A|^2 + m^2 = 0. \end{aligned} \tag{18}$$

The $|G|$ is just the trace of the metric giving us a numerical factor based upon the dimensions of the system . The cross terms canceled out since when we lowered the index on the momentum we pick up a negative, and we are left with what the mass of the charged particle should be based upon the gauge potential. Thus, our constraint equation works by setting the *total* mass to be $m^2 + e^2|A|^2$.

Exercise 2.10: Show that the addition of a two-dimensional cosmological term (2.2.18 on page 18) to the Polyakov action leads to $g_{\alpha\beta} = 0$.

Solution: When the question refers to the cosmological term, the book means this addition to the action,

$$\lambda_1 \int \sqrt{-g}, \tag{19}$$

where we will denote $det(g) \equiv g$. The problem then asks for us to add this to the Polyakov term, find the stress-energy tensor, and find the geometry of the space given by $T_{\alpha\beta} = 0$, but states that this will give us trivial geometry since the equations of motion will imply $g_{\alpha\beta} = 0$. To get started, lets right out the entire action with this new λ_1 term,

$$S = -\frac{T}{2} \int d^2\xi \sqrt{-g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} + \lambda_1 \int d^2\xi \sqrt{-g}, \tag{20}$$

where T is the string's tension, $g_{\mu\nu}$ is the geometry of the spacetime the string is "on" and the $\eta_{\mu\nu}$ is the worldsheet geometry of the string. However, to make calculations less complex, we can let $T = \ell_2 = 0$ for the sake of simplicity.

Now, lets find the stress-energy tensor,

$$\begin{aligned}
\frac{\partial S_p}{\partial g^{\alpha\beta}} &= \int d^2\xi (\delta\sqrt{-g}) g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} + \int d^2\xi (\delta g^{\alpha\beta}) \sqrt{-g} \partial_\alpha X^\mu \partial_\beta X_\mu + \lambda_1 \int (\delta\sqrt{-g}) d^2\xi \\
&= \int (-\frac{1}{2} g_{\alpha\beta} \sqrt{-g} \partial g^{\alpha\beta}) g^{\gamma\delta} \partial_\gamma X^\mu \partial_\delta X_\mu + \int d^2\xi \partial g^{\alpha\beta} \sqrt{-g} \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{\lambda_1}{2} \int g_{\alpha\beta} \sqrt{-g} \partial g^{\alpha\beta} d^2\xi \\
&= \int d^2\xi \sqrt{-g} \left[\partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2} g_{\alpha\beta} g^{\gamma\delta} \partial_\gamma X^\mu \partial_\delta X_\mu - \frac{\lambda_1}{2} g_{\alpha\beta} \right] \delta g^{\alpha\beta} \rightarrow \\
T_{\alpha\beta} &= \partial_\alpha X \cdot \partial_\beta X - \frac{1}{2} g_{\alpha\beta} g^{\gamma\delta} \partial_\gamma \cdot \partial_\delta X - \frac{\lambda_1}{2} g_{\alpha\beta}
\end{aligned} \tag{21}$$

In this first line, this is just taking the variation of the entire integral. Next is recalling what the variation of the weight is with respect to $g^{\alpha\beta}$, and since we would have reused two dummy indices, we needed to replace the indices on the metric of the first term to keep notation consistent. The third line is bringing everything together, and lastly we pulled out the final equation. We also simplified the notation in the last line since the string's coordinates described by X^μ were summed over, they resembled the typical dot product of two "velocities."

Now that we have our stress-energy tensor, we can set it equal to zero, and simplify. Remember, we are looking for a way that would contradict with the statement, "the geometry is unique," or simply a way that would show, $g_{\alpha\beta} = 0$. For this, we set $T_{\alpha\beta} = 0$, and simplify as far as we can:

$$\begin{aligned}
&2\partial_\alpha X \cdot \partial_\beta X - g_{\alpha\beta} g^{\gamma\delta} \partial_\gamma \cdot \partial_\delta X - \lambda_1 g_{\alpha\beta} = 0 \\
&2 g_{\alpha\beta} \partial^\beta X \cdot \partial_\beta X - g_{\alpha\beta} \partial^\delta X \cdot \partial_\delta X - \lambda_1 g_{\alpha\beta} = 0 \\
&g_{\alpha\beta} (\partial X) - \lambda_1 g_{\alpha\beta} = 0
\end{aligned} \tag{22}$$

The first line is simply multiplying by two, the second line is pulling out the metric on the first term to be able to have a summation over the β index. This allowed us in line three to subtract the two quantities, which gives us the issue we are looking for: the metric can simply be canceled since the other two quantities are scalars, which means $g_{\alpha\beta} = 0$, giving us trivial geometry (meaning an action/theory that is not useful).

Exercise 2.12: Use the oscillator expansions in equation (2.3.6 on page 23) and the PB (2.3.32 on page 27) to derive (2.3.33).

Solution: The equations the problem is referring to are equation 2.3.6:

$$X_L^\mu(\tau+\sigma) = \frac{x^\mu}{2} + \frac{\ell_s^2 p^\mu}{2}(\tau+\sigma) + \frac{i\ell_s}{\sqrt{2}} \sum_{n\in\mathbb{Z}-\{0\}} \frac{\alpha_n^\mu}{n} e^{-in(\tau+\sigma)},$$
$$X_R^\mu(\tau-\sigma) = \frac{x^\mu}{2} + \frac{\ell_s^2 \bar{p}^\mu}{2}(\tau-\sigma) + \frac{i\ell_s}{\sqrt{2}} \sum_{n\in\mathbb{Z}-\{0\}} \frac{\bar{\alpha}_n^\mu}{n} e^{-in(\tau-\sigma)}. \tag{23}$$

These are just the solutions for the equations of motion of the Polyakov string (equation 20 minus the cosmological term) for a closed string that obey the periodicity condition, $X^\mu(\tau,\sigma+2\pi) = X^\mu(\tau,\sigma)$. The second equation the problem refers to is 2.3.32 being,

$$\{X^\mu(\sigma,\tau), \dot{X}^\nu(\sigma',\tau)\}_{PB} = \frac{1}{T}\delta(\sigma-\sigma')\eta^{\mu\nu}. \tag{24}$$

This is the Poisson brackets (PB) for the dynamical variables describing the string, which we previously stated. We then are going to use the dynamics of the string, with the PB relationships to derive the other PB's for the oscillating modes and position/momentum relationship:

$$\{\alpha_m^\mu, \alpha_n^\nu\} = \{\bar{\alpha_m^\mu}, \bar{\alpha_n^\nu}\} = -im\delta_{m+n,0}\eta^{\mu\nu}$$
$$\{\bar{\alpha_m^\mu}, \alpha_n^\nu\} = 0, \ \ \{x^\mu, p^\nu\} = \eta^{\mu\nu}. \tag{25}$$

Ok, with that all out of the way, there are a few ways to tackle this problem. The first is to recall that the modes, position, and momentum are functionals of the strings position and momentum, X^μ and Π^μ. From these, and with the definitions of conjugate momentum and position in terms of their integrals, you can perform a Fourier transformation to get the integral expression of the modes, position, and momentum. Then, use the explicit form of the PBs for Hamiltonian field theory as shown here with the example of the oscillators,

$$\{\alpha_m^\mu, \alpha_n^\nu\} = \int d^d x \left(\frac{\partial \alpha_m^\mu}{\partial X^\beta}\frac{\partial \alpha_n^\nu}{\partial \Pi_\beta} - \frac{\partial \alpha_n^\nu}{\partial \Pi^\beta}\frac{\partial \alpha_m^\mu}{\partial X_\beta}.\right) \tag{26}$$

There is also a second method to find the PBs. If we use the relationship between quantum and classical mechanics, $\{,\} \to i\hbar[,]$, then we only need to find the commutator, and drop the $i\hbar$ term. With this, we will then have a linear expansion of PBs, from which we can match and form PBs when expanding the PB of equation 24. Then, after matching coefficients, we can use the fact that if two functions are equal, their Fourier series' are equal as well. This is because the Dirac delta of equation 24 can be expressed as,

$$\delta(\sigma-\sigma') = \frac{1}{\pi}\sum_{n=-\infty}^{\infty} e^{2in(\sigma-\sigma')}, \tag{27}$$

since for the closed string, we have periodicity conditions for the boundary. So, the first method looks long and tiresome, but the second method seem somewhat shaky since we are using some of the knowledge already derived from the longer method to solve the problem. So, we will go through the first method instead.

The operator equations for the modes, position, and momentum will be,

$$x^\mu = \frac{1}{2\pi}\int_0^{2\pi} \left(X^\mu - \frac{\tau}{T}\Pi^\mu\right) d\sigma$$
$$p^\mu = \int_0^{2\pi} \Pi^\mu d\sigma$$
$$\alpha_n^\mu = \frac{e^{in\tau}}{4\pi\ell}\int_0^{2\pi} \left(\frac{1}{T}\Pi^\mu - inX^\mu\right) e^{-in\sigma} d\sigma$$
$$\tilde{\alpha}_n^\mu = \frac{e^{in\tau}}{4\pi\ell}\int_0^{2\pi} \left(\frac{1}{T}\Pi^\mu - inX^\mu\right) e^{in\sigma} d\sigma, \tag{28}$$

Now, lets plug position and momentum operators by replacing the oscillator modes of equation 26 to find,

$$\{x^\mu, p^\nu\} = \{X^\mu, \Pi^\nu\} + \{\Pi^\mu, \Pi^\nu\} = \eta^{\mu\nu}, \tag{29}$$

where the first term is based on the definition of PBs in Hamiltonian field theory, and the second goes to 0. For the oscillator modes we get,

$$\{\alpha_m^\mu, \alpha_n^\nu\} = -im \int_0^{2\pi} \eta^{\mu\nu} e^{-i\sigma(n+m)} d\sigma = -im\delta_{n+m,0}, \tag{30}$$

where we find the definition of the Dirac delta with a cyclic boundary, and the "x" value was 0 (simply plug in the oscillator modes into equation 26 and simplify).

3 Chapter 3: Quantization of Bosonic Strings

Exercise 3.1: Using the commutation relations of the oscillators, carefully calculate the commutator of the Virasoro operators in (3.1.9 on page 33). Show that the normal ordering constant a is formally equal to,

$$a = \frac{D-2}{2} \sum_{n=1}^{\infty} n. \tag{31}$$

Regularize this divergent sum using the ζ-function regularization,

$$\sum_{n=1}^{\infty} n = \zeta(-1) = -\frac{1}{12}. \tag{32}$$

Solution:

Exercise 3.5: Show that only when the left and right intercepts are the same in the closed string, $a = \bar{a}$, can one obtain a nontrivial spectrum consistent with Lorentz invariance.
Solution:

Exercise 3.6: Consider the massive states of the bosonic open string in $d = 26$ at the second level. Show that they form representations of the massive little group $O(25)$.
Solution:

Exercise 3.13: Consider the two-dimensional torus. Find explicitly the two conformal Killing vectors. Find also the two (real) Teichmuüller parameters.
Solution:

Exercise 3.19: Use the BRST transformations (3.8.1 on page 46) to derive the BRST current in (3.8.13).
Solution:

Exercise 3.20: Show by direct calculation that for the bosonic string, $Q_B^2 = 0$ when $d = 26$.

4 Chapter 4: Conformal Field Theory

Exercise 4.1: Use the infinitesimal conformal transformations (4.1.8) to derive the algebra of the conformal group (4.1.11 on page 54).
Solution:

Exercise 4.9: Start from the Euclidean massive scalar propagator in two dimensions,

$$\Delta_F(x,y) = \frac{\ell_2^2}{2\pi} \int d^2p \frac{e^{ip\cdot(x-y)}}{p^2+\mu^2}, \tag{33}$$

and take the $\mu \to 0$ limit to obtain (4.7.2 on page 65).
Solution:

Exercise 4.10: Show that,

$$\partial\bar{\partial} log|z|^2 = \partial\frac{1}{\bar{z}} = \bar{\partial}\frac{1}{z} = 2\pi\delta^{(2)}(z), \tag{34}$$

either by using Stoke's divergence theorem or by regulating the singularity at $z = 0$ and then removing the regularization.
Solution:

Exercise 4.20: Show that $\hat{T}$ defined as in (4.9.3) transforms under conformal transformations as in (4.9.4 on page 69).
Solution:

Exercise 4.31: Derive the equations of motion from the action (4.11.6 on page 74). Show that they imply $\bar{\partial}J = \partial\bar{J} = 0$ when $\lambda^2 = 4\pi/k$,
Solution:

Exercise 4.36: Using the mode expansion (4.12.15 on page 78), and the commutation relations (4.12.16) and (4.12.31 on page 80) show that the two-point function of fermions in the R vacuum is,

$$G_R^{ij}(z,w) = \left\langle \hat{S} \middle| \psi^i(z)\psi^j(w) \middle| \hat{S} \right\rangle = \delta^{ij} \frac{z+w}{2\sqrt{zw}} \frac{1}{z-w}. \tag{35}$$

Solution:

Exercise 4.40: Use the same procedure as that used in the $\mathcal{N} = (2,0)_2$ superconformal case to show that for unitary $\mathcal{N} = (4,0)_2$ superconformal primaries in the NS sector, $\Delta - j \geq 0$, while in the R sector, $\Delta \geq k/4$.
Solution:

Exercise: 4.53: Derive the fermion propagators in the R sector on the disk.
Solution:

Exercise 4.69: Derive the two-point function for a massless scalar on the torus. From this, derive, by involution, the scalar propagator on the Klein-bottle, cylinder, and Möbius strip.
Solution:

5 Chapter 5: Scattering Amplitudes and Vertex Operators

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6 Chapter 6: Strings in Background Fields

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7 Chapter 7: Superstrings and Supersymmetry

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8 Chapter 8: D-Branes

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9 Chapter 9: Compactification and Supersymmetry Breaking

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10 Chapter 10: Loop Corrections to String Effective Couplings

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11 Chapter 11: Duality Connections and Nonperturbative Effects

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12 Chapter 12: Compactifications with Fluxes

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13 Chapter 13: Black Holes and Entropy in String Theory

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14 Chapter 14: The Bulk/Boundary (Holographic) Correspondence

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15 Chapter 15: Applications of the Holographic Correspondence

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16 Chapter 16: String Theory and Matrix Models

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References

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[3] Riley, Hobson, and Bence, *Mathematical Methods for Physics and Engineering, 3^{rd} edition.* (Cambridge University Press, 2006).