Learning General Relativity as an Undergraduate

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Abstract

This article is composed to focus on undergraduates who tend not to have typical opportunities of larger universities, or simply desire to learn general relativity on their own (with the proper background). This is from the perspective of a fellow undergraduate, coming from similar circumstances. Presented will be an outlined schedule for a 14 week semester for an independent study.

1 Introduction

General Relativity captures the interest of many undergraduate students who either wish to learn more about the theory itself, or have desires to pursue the theoretical side of physics. In particular, I fell into both categories; I held the desire to learn more instead of typical popular science talks about General Relativity, and desire, til this day, to work in theory. However, not everyone has the same opportunity, and to those who wish to 'break forth' from those limiting educational gates, this article trends towards you.

An issue that faces many people learning General Relativity (in general), are the mathematical techniques behind the construction of Relativity. This however can be slightly diverted through an independent study since it leaves more time to talk, discuss, and practice such techniques, (thus presenting a positive towards learning on your own). Although I do not condone learning Relativity *strictly* on your own, but instead with the proper authority i.e setting up an independent study with a Professor. How this is resolved is discussed below.

I will first outline the modern day necessity of learning Relativity for any undergraduate with a focus on theory along with common issues. Next will be how the mathematical topics can be surmounted as an undergraduate with the unique text selection. Finally, a structure scheduled relating to a 14 week semester will be laid out with: chapters to read, recommended problems, further resources.

2 General Relativity: a Modern Requirement

General Relativity has become an increasingly necessary subject to learn in graduate school, or at least the mathematical techniques behind the theory have slowly crept up in a necessary to learn more. Such subjects are relativistic astrophysics and cosmology (of course), theoretical particle physics, unified field theory, superconductors (holographic), gravitational wave detectors, and I am certain many more fields. In many of these fields, the 'nitty-gritty' details are not always necessary to master nor learn.

Common issues I have faced, since I can only speak upon my own experiences, was not learning the material or computations, but in recalling the geometry of the symmetries used in tensors of higher ranked. Specifically that of finding the components of the Riemann tensor; such as the 21 independent components when the metric is diagonal. Referring back to the mathematics, the outline listed with the texts take a very cautious, and thorough teaching of Differential Geometry. Of course, one could learn Differential Geometry separately through various texts (link) (which will be explained more below under Resources), but I would argue this is not necessary for an immediate usage of the theory. In general, I would suggest that as long as one has surpassed and completed with an absolute understanding of Calculus III, Differential Equations, and a course in Linear Algebra, serves as great foundations for beginning to learn General Relativity.

Continuing forward, the next section will be a complete regard for how the mathematics can be learned with tips from myself, as well as an analysis of one of the textbooks to be used, *A General Relativity Workbook* by Moore.

3 Mathematics of General Relativity

Differential Geometry is difficult until you stop calling the subject-*Differential Geometry*. This is the fresh of breath air that is found throughout Moore's Relativity textbook. Moore introduces the basics, comparable with Quantum Field Theory textbooks, with 'real' physics every step of the way (where it is applicable): Special Relativity, into 4-vectors, into index notation (I say 'real' since several times within Relativity textbooks all one finds is mathematics at times). Next, what follows is a superb build-up to *The Tensor*. Never once was a line read that goes, "Suppose on some manifold M....." This is what makes Moore very powerful for independent learning.

Although, there are certain times when one must recall cold, abstract truths from Differential Geometry, which is the reasoning behind the independent study being taught with a second textbook: *Spacetime and Geometry* by Sean Carroll. This textbook presents to full extent the mathematics needed to comprehend most (if not all) aspects of Relativity in the most elegant, and thoughtful way I have yet to read.

The hope is that these two textbooks will be presented in a way that seems 'continuously smooth' learning path; with a slight emphasis of material coming form Moore. Where Moore may leave a few details, then Carroll shall take over the fill the gaps. Towards the end of schedule, where we touch upon the Lagrangian formulation, we will solely be relying upon Carroll however.

Next, we will finally view and explain throughout simply how one may learn General Relativity as an undergraduate through an independent study in a 14 week course (1 semester).

4 The 14 Week Course

14 Week Learning plan for General Relativity						
Week	Mon.	Tues.	Wed.	Thurs.	Fri.	Problems
1	Ch.1, Moore	-	M: Ch.2	-	Carroll: Ch 1.4	M: 1.3, 1.6, 2.1
2	M: Ch.2	-	M: Ch.4	-	C: Ch.1.9	M: 4.5, 4.1, 4.8, 3.1, 3.9
3	M: Ch.5	-	M: Ch.6	-	C: Ch 1.4-6, 2.3, 2.4	M: 5.4, 5.7, 6.9, C: 1.7
4	M: Ch.7	-	Q's	-	Q's	M: 7.1, 7.8, C: 1.10
5	M: Ch.8	-	C: Ch.3.4	-	Q's	M: 8.2, 8.6
6	M: Ch.9	-	Q's	-	Q's	M: 9.5, 9.6, 9.8
7	M: Ch.10	-	C: Ch.5.4	-	Q's	M: 10.4, 10.7, 10.10, C: 5.3
8	M: Ch.12	-	Q's	-	Q's	M: 12.1, 12.6, 12.4
9	M: Ch.17	-	C: Ch.3.2	-	C: Ch.3.3	M: 17.3, 17.4, 17.10, C: 3.3
10	M: Ch.18	-	C: Ch.3.7	-	Q's	M: 18.6, 18.8, C: 8(a,b)
11	M: Ch.19	-	C: Ch.3.7	-	Q's	M: 19.1, 19.6, 19.9, C: 3.11
12	M: Ch.20	-	C: Ch.4.6	-	Q's	M: 20.4, 20.6, 20.7, C: 1.8
13	M: Ch.21	-	M: Ch.22	-	C: Ch.4.1-3	M: 21.2, 21.8, 22.6
14	M: Ch.23	-	C: Ch.1.10	-	C: Ch.4.3	M: 23.4, 23.5, S: 4.1

For context: M: ≡ Moore, and C: ≡ Carroll, and problems are labeled similarly, M1.1 is problem 1 from chapter 1 of Moore, and similar for Carroll. This is a three day schedule, which correlates with a 3 credit course. If there are any " Q's ", this simply refers to days that are empty, which can be utilized either as: 'catch-up', a day for questions, extra material, spread out the previous material. Next is the reasoning and layout of the schedule.

Firstly, what is not on the weekly outline are the *Boxes* from Moore's text. These should be thought of as 'entry' and 'exit' tickets per class period/meet-up to reinforce what was just learned, and give a first attempt computation as to how the mathematics are used and physically mean (in the context of Relativity). Again, at least all of the *Boxes* should be attempted during each chapter from Moore.

The first week is a review of Special Relativity and introduction to General Relativity topics. The second week is transitioning from basic Special Relativity notion to 4-vectors and Einstein summation. Third are chapters unique to Moore that go in depth with index notation and special types of tensors (metric, knoecker delta, etc.). Thirdly, this week introduces arbitrary coordinate basis' that construct or lead-to the general metric $g_{\{}mu\nu$,the types of tensors that arise from differential geometry (dual vectors, covariant, contravariant), and how tensors are multiplied. The fourth week is an application of the current knowledge with constructing Maxwell's equations in a covariant form. As a brief note, Carroll does have a solid section on Maxwell, but only after about differential forms. The fifth week looks at the geodesic equation, but from Moore's perspective, he first delivers this equation from the action, not the metric connection; hence, Carroll's section is not within this week. The sixth week gives a physical application to the geodesic equation, this being the Schwarzchild metric. The seventh week, again has similar reasoning as previous, but for particle orbits. The eighth week, is again similar, but for photon orbits and learning about the parameter b. Our ninth turn finally begins to become interesting, with displaying the absolute gradient in Moore's terms (the covariant derivative) leading to Christoffel symbols. Carroll introduces the Christoffel symbols not as a correcting term to the derivative of a tensor, but a metric connection on a manifold. Coming in tenth is calculating the geodesic deviation, thus introducing the Riemann tensor. Eleventh becomes an in-depth analysis of the Riemann tensor. For number twelve, the stress energy tensor. Thirteenth finally constructs the Einstein tensor and equation with interpretation by weak-field limit. And for our final week, briefly is shown how the Schwarzchild solution comes about, or how Einstein's equation is used, and finally how to derive Einstein's equation and various stress-energy tensors are found using the variational principle.

Everything listed is quite "normal" for a first introduction to General Relativity, but perhaps not for the opportunity to only have one course. The only missing points which were not discussed nor added are: Killing vectors, Manifold theory, computational methods, cosmology, black hole solutions gravitational waves (calculation and measure) relativistic stars, experimental tests. Resources (chapters/texts/articles) that I know of that teach these topics reliably are the textbooks listed below (Moore, Misner Thorne and Wheeler, Carroll) except for computational methods. There are textbooks currently out diving into computational methods, but mostly these come by self-teaching, or second hand.

I would still attempt to 'squeeze' in computational methods though since of its high importance in graduate school and research. This can be done by preference with using a python package, or (personally) through the Mathematica package diffgeo: http://people.brandeis.edu/ headrick/Mathematica/ .

5 Extra Resources and Final Remarks

For concluding remarks, I wish to reinforce that this article is towards undergraduates (or people with simply a desire to learn) who may not have the oppurtunity to learn General Relativity within a classroom setting, but does have the minimal requirements (Calculus 3, Differential Equations, Linear Algebra) and a kindful-helpful-willing Professor to set up an independent study to learn.

Finally, I will list a few more texts and resources to learn General Relativity.

1. *Gravitation* by: Misner, Thorne, Wheeler
    - Not entirely recommened for straight learning, but for those curious to learn more in depth mathematically.
2. *Introducing Einstein's Relativity* by: D'Inverno
    - More blunt mathematically, less physical then Moore, but has a good chapter on the variational principle.
3. *Problem Book in Relativity and Gravitation* by: Lightman, Press, Price, Teukolsky
    - The best way to rigorously practice problems in General Relativity.

4. For the mathematica package: diffgeo.m by: Matthew Headrick

    - Personally, this has been the best, and well constructed way to calculate problems in General Relativity, even when it comes to adding more fields (SU(2), SU(3), and so on).

References

[1] Moore, T.A. *A general relativity workbook.* (University Science Books, 2013).

[2] D'Inverno, R. *Introducing Einsteins relativity.* (Clarendon Press, 2008).

[3] Carroll, S. M. *Spacetime and geometry: an introduction to general relativity.* (Cambridge University Press, 2019).

[4] Misner, Thorne, Wheeler. *Gravitation.* (Princeton University Press, 2017).

[5] Headrick, Matthew. *Mathematica Package, diffgeo.m.* (Mathematica Package, 2015).

[6] Lightman, Press, Price, Teukolsky. *PRoblem Book in Relativity and Gravitation.* (Princeton University Press, 1975).