Introduction to Unification Schemes
Brief Qualitative Understanding into Unified Field Theory

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Abstract

In this presentation, we give a qualitative overview of Unification Schemes from first physical concepts, then the fundamental forces, the particle physics that is associated with the theory, and leading to the possibility of a theory of everything. We give an introduction into the history of when unification of physical phenomenon first began, and differentiate between Grand Unified Theories (GUTs) and Theories of Everything (TOEs). Next, we give a first look into each attempt at GUTs and TOEs. We give a semi-technical introduction into the the foremost prominent attempts at unified field theory that is currently dominating many institutions. And finally, a discussion around the doubt of finding either a GUT or TOE.

1 Conventions and Introduction

In this section, we will cover conventions and definitions used throughout this paper for clarity, and future reference. We will also include some common terms in the field in case they may appear in foreign literature, and a quick reference is needed.

1.1 Conventions and Definitions

1. GUT - (Grand Unified Theory), a field theory that attempts to unify all three nuclear forces (weak, strong, and electromagnetic) into a singular force. Some attribute this also with simply quantizing gravity, but not incorporating the other forces.

2. TOE - (Theory of Everything), a field theory that attempts to explain every aspect of the observable, and testable universe through one main concept or fundamental fact (or postulate).

3. SUSY - (Supersymmetry), postulated symmetry between bosonic and fermionic fields in quantum field theory and string theory in which each particle has a *super partner* of the opposite type of particle. An electron has a bosonic particle called the *selectron.*

4. QFT - (Quantum Field Theory), the postulate that all particles are excitations of their given field, which incorporates special relativity and rules of quantum mechanics.

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5. MSSM - (Minimal Supersymmetric Standard Model), the incorporation of SUSY into the standard model that considers only the minimum number of new particle states and interactions
6. Phenomenology - the application of theoretical physics towards experimental data by making quantitative predictions based upon known/proven theories. In other words, if we measure a particle's mass to be 125 GeV (the Higgs), then how from theory could we have predicted that?
7. EWT - (Electroweak Theory), the unifcation of the weak nuclear force and electromagnetic force into a single theory (taking SU(2)⊗U(1) gauge groups).
8. Spontaneous Symmetry Breaking - when a highly symmetric state is brought from a high to low energy state, and the symmetry is thus broken (becomes asymmetric). For example, the Higgs Mechanism is one of one such type but specifically for gauge symmetries.
9. LQG - (Loop Quantum Gravity), a GUT that postulates the basis of space is composed of loops called spin networks, which as a collaboration is spin foam. This allows for gravity to be quantized in the normal way that all other QFT's are done.
10. chiral - where an object is not identical to its mirror image. A fermion that is invariant under parity transformations ($f(x) = f(-x)$) is a chiral symmetry.
11. AdS/CFT - a duality from tpe 2B string theory that relates a gauge theory with a gravitational one. The general case is $AdS_5\ X\ S^5 = \mathcal{N} = 4SYM$, but many take the S^5 as the compactified dimensions and are not necessary for first or second order calculations.
12. S-Duality - an equivalence between string theories *and* QFT which relates strongly coupled fields to weakly coupled fields (a strong-weak duality).
13. T-duality - this solely a string duality. This duality, usually in a toroidally compactified theory, that leaves the coupling constant invariant up to a radius-dependent rescaling, thus holds in each perturbation of the theory.
14. Monstrous Moonshine - as a mathematical concept, is the connection between the monster group and modular functions (analytic functions defined in the upper-half plane, usually holomorphic).
15. Landscape - landscape is a general term referring to all the possible false vacua in string theory. The landscape includes all the parameters that govern each possible compactifications. This is where the number 10^{500} which is the number of metastable vacua.
16. CFT - (Conformal Field Theory), a quantum field theory that is invariant under conformal transformations. By conformal, we mean a symmetry that preserves the angle, but not always the lengths involved. Their benefit is that they can sometimes be exactly solvable.
17. Holographic Principle - a supposed property of quantum gravity that states the description (or information) of a volume of space (or description of information in p-dimensions) can be found in 1 lower dimensions or the area (or the information can be encoded onto a similar geometric structure in p-1 dimensions).
18. Kaluza Klein - a classical unified field theory of gravitation and electromagnetism but in 5 dimensions (from the usual 4). A precursor to string theory. This is done by adding one more dimension to the electromagnetic potential coupled to a scalar field. Or just take your metric and go: $g_{\mu\nu} \rightarrow \tilde{g_{\mu\nu}} = g_{\mu\nu} + \phi^2 A_\mu A_\nu$.

19. $\mathcal{N} = 4$ SYM - (Super Yang-Mills), this is specifically SYM with 4 supercharges (from SUSY) and is invariant under conformal transformations. Yang-Mills is just a non-abelian gauge field.

20. Spinors - elements of a complex vector space that can be associated with euclidean space. Think of a vector pointing along a Möbius strip, the vector then constantly "revolves" around the strip and rotates in the process.

21. Symplectic Geometry - branch of mathematics from both differential topology and differential geometry that studies differentiable manifolds that upon them have closed non-degenerate (structure is preserved) 2-forms. Originally came out of Hamiltonian mechanics and analyzing phase-space.

22. Topology - geometric objects that are preserved under a continuous amount of deformations as stretching, twisting, crumpling, and bending (also shrinking and expanding as long as the geometry is smooth, or can be made smooth).

23. Knot Theory - a study of specific geometric structures from topology that are defined as knots: closed loops that are non trivial (so, you can't just cross a string over each other, but must actually be crossed over and closed).

24. *Foam* - if quantum foam, then these are quantum fluctuations of spacetime on extremely small scale due to QM (idea by John Wheeler). If spin foam, then these are topological structures that are 2 dimensional faces which are a configuration required by the functional integration to obtain Feynman's path integral (think of dynamical 2D structures that create spacetime).

25. Riemanian Geometry - branch from differential geometry that studies Riemannian manifolds, or in other-words, manifolds that are smooth (infinitly differentiable) and can be described by a mathematical object called the metric.

26. Metric - (as a tensor) a geometric function that has 2 inputs being specifically tangent vectors at a point on a surface and produces a scalar ($g^{(2)}$[slot 1,slot 2]$\rightarrow \phi$(scalar/number)).

27. SUGRA - (supergravity), a field theory that combines supersymmetry and general relativity. SUGRA is the gauge theory of local supersymmetry. Although the original formulation is not used, many branches and fixes from the original are still widely used.

28. ADM Formalism - (Arnowitt-Deser-Misner), a Hamiltonian formulation of general relativity that is extremely crucial towards the canonical (momentum and position) quantization of gravity, and extremely useful in numerical relativity by splitting the 4D manifold into the 3D manifold that has a null vector leading to the following 3D manifold (stacks of 3-manifolds each with a null vector pointing to the next). One benefit in general is that the energy of the gravitational system is taken to be the strength of the energy at infinity (it's useful for computing boundary terms in gravitational partition functions).

29. Holomorphic - complex-valued (analytic) function of one or more complex variables, or at every point of the function's domain, that is complex differentiable in a neighborhood (the poles are either nice, or easy to get rid of). The big thing is that a holomorphic function is left moving since the continuation of the field goes as $\sigma_0 - \sigma_1$ and antiholomorphic are right-moving and goes as $\sigma_0 + \sigma_1$.

30. Connections - (as a mathematical term), describe the relationship of geometric object with the manifold it resides on and tells you how it changes with respect to it. For example, if our geometric object in question is a vector, then the connection of the vector would tell you how it changes as it moves along the given manifold. If the manifold in question is a sphere, then the connection would tell you that the angle of the vector changes as it moves along a sphere.

31. Ashtekar's variables - new set of canonical variables to rewrite the metric-canonical variables on the three dimensional slices made by the ADM-formalism.

32. Weinberg-Witten Theorem - a statement that massless particles with spin, $s > \frac{1}{2}$ cannot carry a Lorentz-covariant current (invariant under lorentz transformation that is covariantly described) while massless particles with spin $s > 1$ cannot carry a Lorentz-covariant stress-energy. This means that we can not make a graviton out of two spin 1 gluons since the graviton is a spin 2 particle.

33. RR-Charges - RR stands for Ramond-Ramond, these are the general differential forms in 10 dimensions that can describe forces, these are mainly found in type 2 supergravity formulations. An example is the p-form generalization of electrodynamics.

34. Dilaton - hypothetical scalar-particle that only appears in theories with extra dimensions **and** when the volume of the extra dimensions that is compacitified varies. An example is Brans-Dicke theory, but the potential is defined explicitly as a dilaton.

35. geometric flow - (concept from differential geometry), the generalized gradient flow of a functional on a manifold that can be either intrinsic (no background embedding) or extrinsic (manifold embedded). The difference between a gradient flow and the typical gradient, is that one is continious over an evolved parameter, and the other is discrete.

36. analytic - generally a function that locally appears to be expressed as a convergent power series. Now, this is usually in the complex plane, which then gives rise to holomorphic functions. This is generally the context of which texts speak of (analytic is just a broader term then holomorphic).

37. decoherence - (quantum), loss of quantum coherence.... Well of course, but a system has coherence if there is still a phase relation between different states of the wavefunction. The apparent losing of this, now labels the system as decoherence, or we just lost a lot of information about the wavefunction, since we just measured it.

38. Firewall - (black hole), the idea that postulates of black-hole-complementarity (information is reflected at horizon and passes through and cannot escape plus no observer can confirm both stories) are not self-consistent One of the postulate put forth is that the radiation of a black hole must not be in a mixed state.

39. Information - (black hole), simply, the amount of yes/no questions it takes you to know everything about a system. The yes/no's are 0 and 1's, and the questions are equations probing the system of 1's and 0's. The current issue is that it apparently appears that physical information can permanently leave this universe after is passes through the horizon of a black hole, and seemingly violates the idea of predictability in the wave function, since it must now stop *somewhere* (this violation is under the idea of unitarity in quantum mechanics).

40. Entropy - (black hole), the apparent relationship (to first order) that the area of a black hole is proportional to the entropy. This is due to the second law of thermodynamics.

41. Evaporation - (black hole), also called Hawking Radiation, the concept that black holes lose energy with respect to the universe by irradiating out particles, and thus losing mass.

42. CPT Symmetry - absolutely fundamental symmetry of every physical law that is a Lorentz invariant QFT described by a Hamiltonian. CPT stands for Charge conjugation, Parity transformation, and Time reversal. Simply put, no realistic field can have asymmetry under any of these three.

43. compactify - to reduce the number of dimensions in a physical theory by incorporating, or *wrapping* the extra dimensions into a compact shape such as a torus, or a sphere. The most basic example is taking a long open cylinder, applying Kaluza-Klein decomposition, and now having an effective theory over what appears to be a single-dimension string.

44. grand unification - the idea that very early on in the universe, each force unified into a single super force. More mathematically, each group description of the fundamental forces formed a super group: $SU(3) \otimes SU(2) \otimes U(1)$. This is usually done at around $10^{16} - 10^{19}$ GeV.

45. Bianchi Identities - the reason we can ensure a vanishing covariant derivative of the stress-energy tensor. But also, more importantly, if you can construct a tensor that obeys a Bianchi identity, then that tensor can be physically interpreted with an associated stress-energy tensor. The idea originally came from taking the boundary of a boundary, which gets you zero.

46. diffeomorphism - an isomorphism (relating two structures of the same type that have an inverse, the metric is isomorphic, meaning you can move the metric and distances will remain the same), or a map (associates unique objects to every element in a set/function) between manifolds that is differentiable and differentiable inverse

1.2 History

The history of unified field theory can be messy, but is best described, and marked, by revolutions, or major concepts that arrived in the field. Even though the field was not formally described really until Dirac came around, we can still trace back the first definition of it.

The first instance of simply bringing two physical concepts together is one I think everyone knows, which is Maxwell saying $\vec{E} + \vec{B} \propto \textit{Electromagnetism}$. This being done in 1865 by Maxwell was the cornerstone of the mathematical framework, but many other people added to his work such as: Ørsted, Hertz, Heaviside, and Faraday[1].

The next instant is the most famous, when Einstein brought together both space and time into spacetime. This was the bedrock towards formulating special relativity, along with Lorentz coordinate-frame transformations. The unrecognized aspect of relativity though is what it sparked among mathematicians: Poincaré derived his transformations and geometric view of relativity, Minkowski (one of Einstein's previous professors) derived Minkowski spacetime, and Lorentz and Poincaré began formulating the tensorial version of special relativity (which laid the framework for Einstein).

Einstein also formulated gravity from special relativity, which led to general relativity. One could say he unified gravity with spacetime, and recognized they are the same thing (in a sense). His equation, $G_{\mu\nu} = T_{\mu\nu}$ is relating the curvature of spacetime (making special relativity covariant) to the matter in said spacetime (or given scenario). It should be noted that the formulation of general relativity came from the mathematical work of Bernhard Riemann, which was introduced to Einstein by his friend Grossmann[1].

Next in unifying physical concepts was Paul Dirac who brought together special relativity, and the quantum theory of the electron, or the Schrodinger equation. Dirac's equation, $(i\gamma^\mu\partial_\mu - m)\psi = 0$ was originally proposed to describe the electron, but we now know it describes all fermions (at least their particle, not force incorporation). Although before Dirac, the first attempt at quantizing the electron was the Klein-Gordon equation, $(\partial_\mu\partial^\mu + m^2)\psi = 0$, which does not describe the electron, but instead particles with no spin (like the pion).

Now, to talk any further on unification, it can become *quite* messy, since there really were not any true unifications occurring for the next fifty years, but mainly new formulations of fundamental forces such as electromagnetism (plus with the dirac equation and a gauge-fixing term gives QED), the discover of the SU(2) group being the underlying formulation of the weak force (QFD), and SU(3) group for the strong force (QCD). All of this constituting the construction of quantum field theory over about 30-50 years[2].

However, in 1964, Salam, Glashow, and Weinberg in order to solve parity violation issues in the weak force interactions, realized that if the weak and electromagnetic force are actually unified under weak isopsin fields and weak hypercharge fields, then they produce a symmetry which can be broken. This symmetry was predicted to be broken under a specific field, a gauge-boson field, being the Higgs[3,4]. This new theory was called Weinberg-Salam Theory, or simply electroweak theory, which is the main takeaway for us, not the new particles or spontaneous symmetry breaking, which is extremely important, but the concept that fundamental forces apparently unify under high energy conditions.

But now in our history, we haven't mentioned gravity for what is about 60 years. However in the background of the particle fever[5], and successful formulations of quantum field theory, there were a few attempts at quantizing gravity. The first was very early on by Bronstein in 1930's who was the first person to face an infinity with quantum gravity. Sadly, he lived in Russia in the 1930's when the USSR did not exactly like free-thinkers, and was killed in 1938. He did introduce the notion that the fundamental theory of quantum gravity should only include the constants c, G, and h [6].

To talk further about unifying gravity and quantum mechanics, really only deals with string theory, which is a TOE not a GUT, and a few instances of LQG and gauge theory formulations. From 1943-1970, the S-matrix, Regge Calculus, and Bootstrap models were developed. From 1970-1974, the hint that the strong force is mediated by a string instead of gluons was postulated, but SU(3) formulation predictions were better. Between 1974-1984, John Schwarz and Joel Scherk discovered a spin-2 boson from their bosonic string theory, but ran into issues[7]. In 1984-1995, Schwarz and others incorporated SUSY into their bosonic string theory and came up with Superstring theory, the first revolution. Then the second superstring revolution occurred being M-Theory, F-Theory, Branes, and the AdS/CFT duality being the victors of 1994-2003. What I glossed over though was the formulation of LQG between 1986 and 1994 by Ashtekar, Jacobson, Rovelli, and inadvertently Roger Penrose[8].

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[1] The concept of spatial curvature was originally started with Gauss, and then Riemann generalized the concept.

[2] I would like to point out though that the formulations for each theory had many infinities, especially in the 1920's until '30's, and everyone was worried, until a whole 20 years later that renormalization came around to save the day.

[3] The basic of how the Higgs Mechanism works is that if we start at high energy, then we evolve in time towards a lower state of energy, the symmetry is broken because our potential moved from $V(\phi) \to V(\phi + V(0))$.

[4] Electroweak theory was shown to be renormalizable seven years later by 't Hooft

[5] This is the name of an actual documentary that is pretty good.

[6] C from special relativity, G from gravitation, and h from quantum theory

[7] Issues such as having tachyons *everywhere* in the theory, and it required 26 dimensions!

[8] Penrose devloped something called spin networks which represents states and interactions between particles

To wrap-up, the best way to view the history recently described is by a diagram I found on wiki which incorporates every aspect we described here, except for the formulation of special relativity and QM into the Klein-Gordon and Dirac equation.

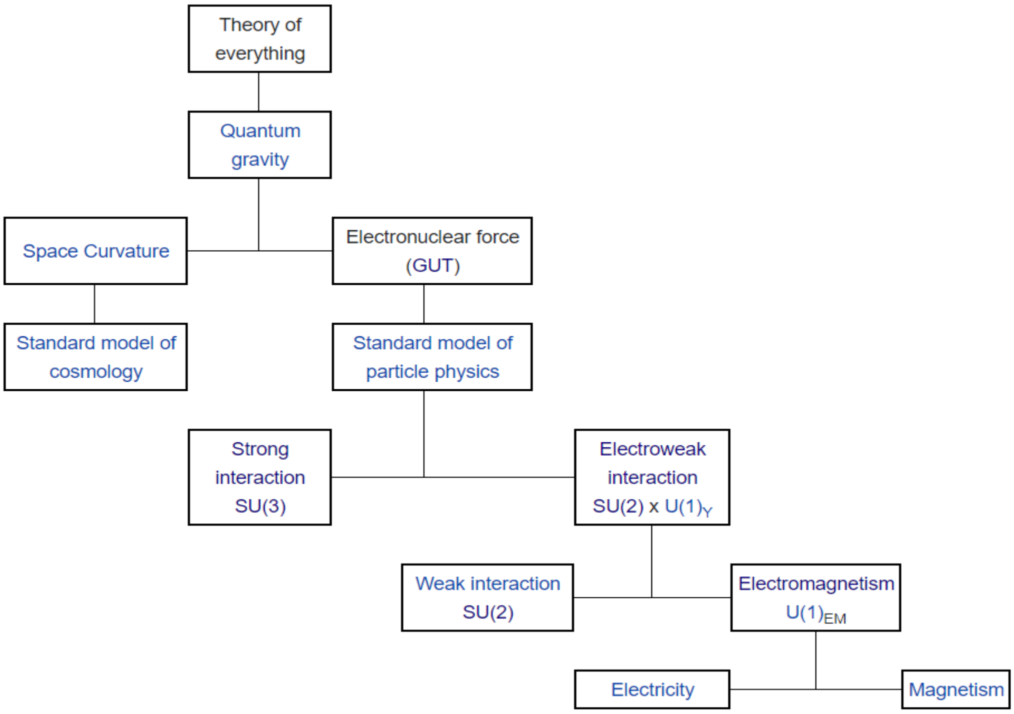

Figure 1: Diagram of unification of forces via fundamental forces.

2 Current Efforts

Next, we shall introduce a few of the most active unified field theory efforts that are occurring at about every institution; some more and less then others (and perhaps some not even talked about)[9]! These efforts being string theory, loop quantum gravity, extensions of the standard model, emergent spacetime, and gauge theory.

2.1 String Theory

String theory in general is the realization that if we extend a particle's worldline by one dimension, we form a sheet, or a worldsheet, which then has the properties, classically shown, of a vibrating string through spacetime. We will briefly visit each derivation of string theory.

2.2 Bosonic String

The most talked about, perhaps due to how creative and exotic the theory is, theory of everything is string theory. It was originally thought of as a theory to describe the strong interactions between particles, and can still describe the interaction in a specific regime, but grew to become something more.

The first true string theory was bosonic string theory developed in the late 1960's and, as you could guess, only described bosons. The big *hype* that the bosonic string generated is its ability

and fields in QM which is crucial in LQG.

[9]An example of this is 2-Time physics, or 2T physics. The forefront is being led by Itzhak Bars at the University of Southern California, and has seen some *results*.

to predict the graviton. However, every version of the bosonic string, (open, closed, oriented, unoriented) had tachyons[10]. However, the way the bosonic string was formulated is what was crucial; the construction from a worldline to now a worldsheet, and in turn formulating a new metric. However, the bosonic string did carry with it, as all stirng theories do, is that of extra dimensions, and in particular, this one has 26 (spatial + temporal := 25+1).

The fully bosonic string quantized through the path integral is done via the Polyakov action with $G_{\mu\nu}$ the metric and $x^\mu(\xi)$ the worldsheet. The reason for Greek and Latin indices is that if a geometric object has Greek indices, then it belongs on the metric, and we are referencing the spacetime, if a Latin index, then we are referring to the geometry of the string's metric. But if we see a quantity that is the derivative of the spacetime with respect to the string, then we are looking at interactions and how it behaves in said spacetime (the string lives on the background called spacetime).

$$S[g, X] = \frac{T}{8\pi} \int_M d^2\xi \sqrt{g} g^{mn} \partial_m x^\mu \partial_n x^\nu G_{\mu\nu}(x). \tag{1}$$

2.3 Type 1 & 2 String Theory

Type 1 and 2 string theories are string theories that took the bosonic string and SUSY and combined them together. Type 1 string theory contains both open and closed strings and does not require and orientation. Then there is type 2 string theory which only include oriented close strings, but have the maximal amount of supersymmetry (32 supercharges). Type 2 is also an umbrella term for Type 2A and 2B string theory with the difference being in their chirality (A is left-right symmetric and B is left-right asymmetric). They both live in 9+1 dimensions.

For reference, the lagrangian density is given for all three. To find their actions, simply put their density inside an integral over the worldsheet with the proper metric weight. The order goes: type 1, type 2A, and type 2B.

$$\mathcal{L}_{type\ 1} = \frac{T}{2} h^{ab} (\partial_a X^\mu \partial_b X^\nu - i\bar{\psi}_\mu \partial \psi^\mu) g_{\mu\nu} \tag{2}$$

$$\mathcal{L}_{type\ 2A} = \frac{T}{2} h^{ab} (\partial_a X^\mu \partial_b X^\nu - 2\psi^\mu_+ (\frac{\partial \psi^\mu_-}{\partial \sigma} - \frac{\partial \psi^\mu_-}{\partial \tau})) g_{\mu\nu} \tag{3}$$

$$\mathcal{L}_{type\ 2B} = \frac{T}{2} h^{ab} (\partial_a X^\mu \partial_b X^\nu + i\bar{\psi}_\mu \partial \psi^\mu) g_{\mu\nu} \tag{4}$$

2.4 Heterotic String Theory

Heterotic strings are closed, and a hybrid of type 2 and bosonic string theories. Now, here is the weird but extremely cool part; we know strings can have left and right-moving excitations, which are decoupled, and thus we can impose the left-moving excitations as the bosonic string in D=25+1 while the right-moving excitations are type 2 superstrings moving in D=9+1 dimensions. However, we are now left with 16 dimensions, which need to be put away somewhere in order to fit the strings on the D=10 supergravity. The 16 dimensions can be compactified in 2 different ways (meaning two different heterotic strings) on the gauge group SO(32) and the other on $E_8 \otimes E_8$.

The lagrangian thus contains the basic string structure from the bosonic string and all 32 supercharges from type 2 string.

[10] Usually, if a theory of particle physics, or physics in general, have tachyons, or faster then light particles, it is a bad sign since they appear to violate the rules of relativity.

$$\mathcal{L} = \frac{T}{2}h^{ab}(\partial_a X^\mu \partial_b X^\nu - \sum_{k=1}^{32}(i\bar{\lambda}_\mu \partial \lambda^\mu) - (i\bar{\psi}_\mu \partial \psi^\mu))g_{\mu\nu} \tag{5}$$

2.5 D-Branes

D-branes are slightly weird objects to talk about, so perhaps the best place to start is where the *D* in D-branes come form. The 'D' stands for Dirichlet, which comes from the specific boundary conditions that strings occur, such as the bosonic string. However, this remained the only insight into this boundary condition for almost 20 years until Polchinski came around in 1989 and recognized their importance. Polchinski saw that the the Dirichlet boundary condition of the string is really a true physical object, and not just a boundary condition. Thus, these new objects can vary in any dimensionality all the way up to 25 for the bosonic string, and hence they are called D-branes. If you hear D0-brane, this is just a single point, a D2-brane is a plane, and a D25-brane is a 25 dimensional object that fills the space of the bosonic string (somehow imagining 25 dimensions...). The also must exists due to something called T-duality.

Now, these branes are usually coupled to a *B-field* called the Neveu-Schwarz B-field, and must couple to another confusing topic called Ramon-Ramond charges (fields and potentials included), which can only be done **not** on a worldvolume (an d+1 extension of the worldsheet) but a *worldhypervolume*. This looks something like equation 6, which is the action, but do not worry, most of the time you can estimate a Dp-branes behavior by relating how closely it's structure looks compared to a black hole.

$$S = q_{RR}\int C^{p+1}_{\mu_1 \dots \mu_p}\frac{\partial x^{\mu_1}}{\partial \xi^{a_1}}\dots\frac{\partial x^{\mu_p}}{\partial \xi^{a_p}}h^{a_0 \dots a_p}\sqrt{-det\ h^{a_0 \dots a_p}}d^{p+1}\xi \tag{6}$$

2.6 M and F-Theory

To put is lightly but also as formal as possible, M-theory unifies all consistent versions of superstring theory (everything we just discussed minus the bosonic string) in one dimension higher (so 11). This is done via dualities between type 2A and heterotic strings. A good view of how M-theory relates to the other string theories is in figure 2. M-theory was postulated by Witten in 1995. It should be noted however that a formal derivation or Lagrangian formalism of M-theory has not been written down[11], but M-theory can still be used via the known properties that M-theory holds.

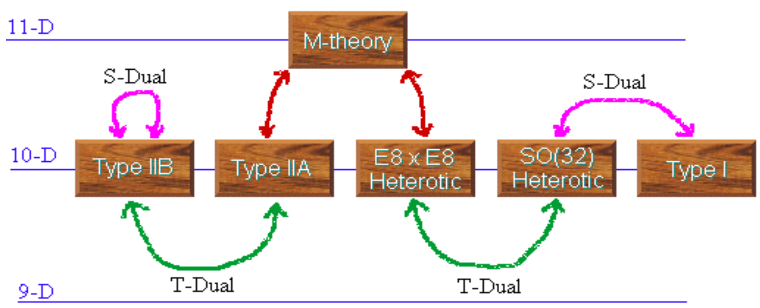

Figure 2: Graphical representation of string dualties with respect to dimensions.

F-theory was developed around 1996 and is a string formalism constructed from type 2B strings in 12 dimensions. However, these 12 dimensions are not 11+1 but instead 10+2, meaning

[11]There are ways to write down a formal lagrangian by making assumptions about the system, but nothing in general has been derived.

there are now 2 time dimensions. Disregarding how mysterious this is, F-theory found its initial success by being compactified onto a torus, thus obtaining type 2B superstring theory. F-theory also finds success in the landscape with $10^{272,000}$ possible vacua, but has found 10^{15} of those to be solutions consistent with the standard model. There has not been a formal lagrangian written down just like in M-theory, but has workable properties.

2.7 AdS/CFT Duality

This is the most worked on topic as of right now in string theory, the relationship derived by Maldacena in 1998 that $AdS_5 \; X \; S^5 \propto \mathcal{N} = 4 \, SYM$. This is a statement that, disregarding the compactified S^5 sphere, gravity on the 5-dimensional AdS spacetime is equal, or *dual*, to the Super Yang-Mills theory with 4 supersymmetries (that's the $\mathcal{N}$=4 part) as a conformal field theory. The actual mathematical relationship, or how we can relate the two is done via partition functions as $Z_{AdS} = Z_{CFT}$ or more formally,

$$< \mathcal{T}(e^{\int d^D x J_{4D} \mathcal{O}(x)}) >_{CFT} = Z_{AdS}[\lim_{boundary} J\omega^{\delta - D + n} = J_{4D}]. \tag{7}$$

From this definition, we can see that the geometry imposed is that we may have any geometrical, as long as inside it is *filled* with gravity on the 5-dimensional AdS, while on the outside it is bounded by the 4-dimensional CFT. Usually, since we are dealing with a conformal field theory, the duality can best be visualized as a cylinder.

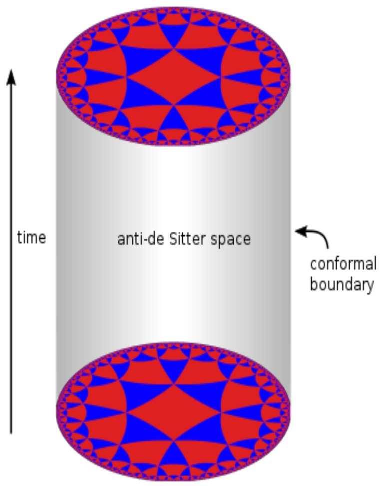

Figure 3: Pictorial representation of the AdS/CFT duality.

As you may have noticed, we are relating a 5-dimensional object towards a 4-dimensional one, which sounds like a holography. This is actually exactly what the duality is, and the first mathematical instance that the universe may really be holographic. Holography is also a supposed property of quantum gravity, making the duality even better looking. Actually, before the duality, t' Hooft postulated that the universe may be holographic by observing the entropy of a black hole is described not by the volume but area as Hawking stated. This postulate was given a precise string interpretation by Susskind.

Since this was the first non-string duality, many more dualities have been postulated such as dS/CFT (useful for cosmology), Kerr/CFT (useful for studying spinning black holes), AdS/CMT (useful in exploring condensed matter, holographic superconductors), and AdS/QCD (useful for strongly coupled systems, such as quark-gluon plasma).

Now, much of the research has been using this duality primarily as a tool to probe the current unknowns in other fields of physics since it seems to relate gravity and matter, almost as a theory of quantum gravity, but gravity is not directly quantized. Due to this, many have begun to pry away from applications, and look more deeply into the actual geometry of the duality. This has specifically attempted at studying the geometry of a black hole past the event horizons, firewalls associated with black holes, and recently been used to solve the Information Pardox of black hole evaporation.

2.8 Standard Model Extensions

Many people do not exactly prove of how far string theory has gone, since it seems extremely un-physical, and can never be tested. Thus, some people have attempted to extend what we know about the standard model out a little further from either implied conjecture, or even taking results from both string theory and loop quantum gravity, and making them fit into the standard model.

There is an attempt to incorporate SUSY into the standard model in various ways. This is mainly in order to cut down on the number of parameters the standard model has (anything down from 20). This is in the form of either the minimal amount of supersymmetries, more constrained, or even adding an extra scalar field. You can also get MSSM from string theory.

People have also began to incorporate operators that break the lorentz symmetry of the system, since this has been found in both string theory and LQG. This is implied by the CPT violation.

2.9 Loop Quantum Gravity

Now, some of the difficulties in quantizing gravity are not just their descriptions, or they both break down at physical singularities, but its also about incorporating known properties of gravity such as background independence and which definition of time do we use. Loop quantum gravity was formulated to fix the issue of background independence[12].

Think of the main issue like this, in QM, the operators we formulate are tied into the equations and functions we use, thus the "matter" and "coordinates" are intertwined. But in gravity, or specifically general relativity formulation of gravity, these are independent quantities (this issue actually becomes worst in QFT).

So, why not try to incorporate the fuzziness of QM into the metric. This is done with the Wheeler-DeWitt equation, but is unsolvable. The equation is presented below for reference with G_{ijkl} being a geometrical object that behaves like a connection coefficient of the three metric.

$$[-G_{ijkl}\frac{\delta^2}{\delta\gamma_{ij}\delta\gamma_{kl}} - R^{(3)}(\gamma)^{1/2} + 2\Lambda\gamma^{1/2}]\Psi[\gamma_{ij}] = 0 \tag{8}$$

Now, what LQG does is take this equation, but runs with it, and makes it more abstract with connection mathematics. Einstein actually tried to do this by reformulating general relativity in terms of only connections, but it didn't work. However, in the 1980's Ashektar achieved this. With this new formalism, Lee Smolin and Carlo Rovelli were able to solve the Wheeler-DeWitt equation with Ashtekar's new connection variables, but with a new trick. They evaluated Ashtekar's variables over closed loops, so each point is connected with each other. Out of this new construction of spacetime, gravity can be formulated, and also quantized (and with background independence)! **This is Loop Quantum Gravity**.

[12]Quantum mechanics is formulated on a flat surface, and thus the particles do not interact with what is "holding it up." But this is exactly one of the hallmarks of gravity, if we put something in the background, the background responds.

The structure of this spacetime is actually called spin networks. There is also a lot of work with trying to incorporate Penrose's Twistor theory in LQG to have a better relationship with today's language since twistors can look like weyl spinors, part of the conformal groups, and be hermitian[13].

But, this is only in 2+1 spacetime, not our observable 3+1. We also have not found the classical limit of of LQG, but the days are early. The big takeaway is that we haven't assumed strings or extra dimensions, we started with fundamental principles, and LQG predicts a variable speed of light based upon the energy of the photon[14].

2.10 Emergent Gravity and Geometrical Quantization

As our final theory of quantum gravity, we will look at briefly constructing gravity as an emergent property from quantum mechanics and thermodynamics, and directly quantizing geometry itself.

When one says *emergent* with respect to gravity, they mean that gravity is simply a force that arises naturally from quantum mechanics. This line of thought came from string theory, black hole mechanics, and quantum information in which the description of gravity comes from the quantum entanglement of small bits of spacetime information. This formulation obeys the second law of thermodynamics (the one where entropy increases as time progresses). This attempt is mainly to understand what quantum gravity really is, and what properties QG holds.

As with respect to quantizing geometry itself, this attempt is with respect of taking ingredients of classical phase spaces from symplectic geometry in order to define a quantum theory. This effort comes from first understanding how we usually quantize. To quantize a theory, we usually describe our theory in terms of a Hamiltonian, and then define the conjugate position and momentum, from which we can define our commutators. But, what if we can describe the Hamiltonian formalism in terms of geometry? This is done with symplectic geometry. There have been attempts to do this with general relativity, but the relationships between symplectic geometry and Riemmannian geometry are still being defined and thought of as we speak.

2.11 Gauge Formulations

The fact that we have been able to successfully formulate a clear description of the electromagnetic, weak, and strong nuclear force using gauge theory would make it seem that gravity can be described by a gauge theory as well. Well, the answer of whether we can or can not is skewed based upon usage. Yes, there is a formulation of *classical* gravitation which looks almost like the tetrad formulation of gravity, but can not be quantized. Then there gauge gravitation theory, which is an extension of Yang-Mills, and is still murky as too whether or not it accomplishes its goal since the diffeomorphism is not clear. However, whenever constructing gravity not via a metric, there is always two glaring issues: diffeomorphism and stress-energy tensor. But, if one can work out the technical details of the gauge formulation (as well as a few infinities here and there), it would appear as a good candidate.

3 Serious Doubts

This section will not be addressing how each individual attempt has specific issues and problems associated with each, but actually addressing whether or not a TOE, or even a GUT can actually

[13]I should mention though that twistors are extremely useful in both LQG, string theory, and even in extensions of the standard model. It's just that when Penrose first formulated them, they were extremely complicated, and didn't seem like the answer.

[14]Gamma rays travel slower while lower energy light rays travel close to the normal speed.

be formulated.

The first idea of doubt comes from **Gödel's incompleteness theorems**. Gödel states that there are inherent limitations of every formal axiomatic system that can model the given arithmetic. In slightly easier English, if we have a theory that literally describes the basics and fundamentals of reality, then the theory will be either incomplete or inconsistent.

Dyson also stated via logic of Gödel's theorem, that pure mathematics is inexhaustible. Thus no matter how many problems we solve, there will always be more. Even Hawking thought along these terms, although initially he did not.

Philosophers and physicists have also began to realize that no physical theory to date is absolutely 100% accurate. Instead, new physical theories simply have more precise approximations, and better successive approximates being more accurate over a larger and larger array of phenomenon. However on the flip-side, people have pointed out and claimed that even though that mathematics is being more complex with each passing theory, the underyling gauge symmetry of each with the number of dimensionless physical constants, the theories are actually becoming simpler. Even then though, the process of simplifying at every single theory can not go on forever[15].

As a final note, Weinberg has pointed out that calculating the precise motion of an actual projectile in Earth's orbit is impossible. Thus, how can we actually know our theory of projectiles is truly accurate? We understand principles well enough, but can not explain them to their fullest extent, and neither will.

Notes and References

For a final sign-off, the references section will be full of both recommendations towards textbooks of subjects to read, as well as technical papers that were discussed throughout. My recommendations of textbooks (and some papers) will be highlighted yellow.

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